DERIVING AUDITORY FILTER CHARACTERISTICS FROM NOTCHED-NOISE MASKING DATA: MODIFIED DERIVATIONS

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DERIVING AUDITORY FILTER CHARACTERISTICS FROM NOTCHED-NOISE MASKING DATA: MODIFIED DERIVATIONS

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Abstract

The notched-noise masking technique and associated methods of analysis have seen wide use in the study of auditory frequency selectivity. Unfortunately, formulae given in the literature for the prediction of detection thresholds for roex filter shapes, and for the calculation of equivalent rectangular bandwidths are in error. The effect of these errors is small for the vast majority of studies on normally hearing listeners but may be serious for studies with hearing-impaired listeners. The errors can be avoided by the use of appropriate limits of integration, as shown in the new derivations given here. It is also shown that the use of equivalent rectangular bandwidth as a summary measure of filter sharpness is often inappropriate (as it is not applicable to a filter shape that is frequently used), and alternatives are suggested.

1. Introduction

The idea that the auditory system performs a kind of frequency analysis on incoming sounds, and thus may be likened to a filter bank, has long been a mainstay of auditory theory. Only relatively recently, however, have there been significant advances made in psychoacoustical methods for determining the shapes of these "auditory filters". Although a variety of experimental techniques has been proposed (e.g., Houtgast, 1974, 1977; Pick, 1980; Zwicker, 1974), the notched-noise masking method and analysis procedure introduced by Patterson and his colleagues has had, perhaps, the most extensive development. It has been used to: investigate changes of auditory filter bandwidth and shape with changes in frequency and level; explain the discrepancies and similarities among auditory filter bandwidths estimated from various methods (e.g., critical ratios, psychophysical tuning curves, and notched-noise masking studies); and, characterise the effects of aging and hearing impairment on auditory filter shapes (see Patterson & Moore, 1986, and references therein).

The basic procedure for a notched-noise experiment is straightforward. In the simplest and most common case, the threshold for a sinusoidal probe tone of fixed frequency is determined in the presence of a number of relatively wide-band noise maskers. Typically, the noise bands are of constant spectral level and have a spectral notch centred on the probe frequency. One noise band without a notch is also used. Thresholds for the probe tone are then plotted as a function of notch width. Although there are a number of variations on this basic design (e.g., the

level of the probe tone can be fixed and the level of the noise varied so as to just mask the probe, the probe can be placed asymmetrically in the spectral notch), only this fundamental case will be discussed.

Much of the power of the notched-noise technique lies in the procedures developed for the analysis of the resulting data. These rely on the *power-spectrum model of masking* and a family of filter shapes with which it is possible to derive a relatively small number of meaningful parameters that describe the empirical data reasonably well. Unfortunately, there are a number of errors in the published derivations of these models which can lead to errors in the estimated parameters, especially when applied to data from hearing-impaired listeners.

2. A tutorial example

Let us consider an imaginary, but fairly typical, notched-noise experiment. The sinusoidal probe is fixed at 1 kHz and the wide-band noise masker has a spectrum level of 60 dB SPL/Hz and extends from 0.2 kHz to 1.8 kHz. Thresholds for the probe are determined for notch widths of 800, 600, 400, 200 and 0 Hz (the no-notch condition). One possible outcome (only likely from a listener with a significant hearing loss at 1 kHz) is shown in Figure 1. Probe threshold decreases slightly as the notch widens with a change of 3 dB from the no-notch condition to the widest notch. Using the procedures detailed in Patterson and Moore (1986), the so-called roex(p) model is fit to the data. This is the simplest filter shape of the roex family with two independent variable parameters: (1) k, a measure of efficiency, and (2) p, a measure of selectivity. Larger p's indicate greater selectivity, hence narrower bandwidths. A completely flat (infinitely wide) filter would be indicated by a p value of 0. The results of this analysis indicate that a roex(p) filter with p = 2.85 fits the data (as shown by the solid line in Figure 1).

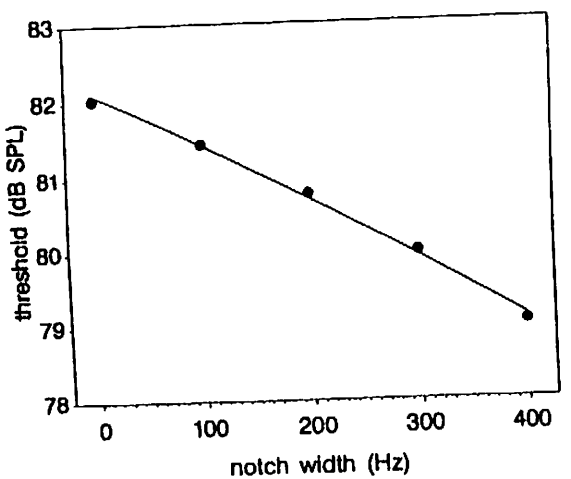

Figure 1. Data from a hypothetical notched-noise experiment (filled circles) and the fit to the data by a roex(p) model (solid line), using the formulae provided by Patterson and Moore (1986).

In short, the analysis procedure claims that this particular listener has some (if severely degraded[1]) selectivity. Consider, however, what result we would expect for a listener with no selectivity whatsoever - that is, with an auditory filter that is completely flat. Because the masking bands of noise have fixed edges (as is commonly, although not uniformly, done), the amount of energy in the noise must necessarily decrease as notch width increases. Therefore if the main assumption of the power spectrum model of masking holds (that the detectability of the probe is determined completely by the signal-to-noise ratio at the output of the auditory filter), then *even a listener with no measurable selectivity will show thresholds that decrease with increasing notch width*. For example, since the noise band here was assumed to be 1.6 kHz wide, then the widest notch of 0.8 kHz will remove half of the energy in the noise, a drop of 3 dB. In fact, the data in this example were generated by a correct roex(p) model with p set to 0 (that is, an infinitely wide auditory filter).

In the next section, I develop the power spectrum model for the roex family of filter shapes so as to generate correct estimates of p for all sets of data. It will be shown that the error in estimation here (and the published derivations this model fitting was based on) arises from the use of incorrect limits of integration. For the same reason, a formula given in the literature for the calculation of equivalent rectangular bandwidth is in error. Fortunately, these errors are only serious when selectivity is severely degraded in comparison to what is typically found in normally-hearing listeners.

3. The power-spectrum model of masking for roex filters

The power-spectrum model of masking, especially as applied to roex filters, has appeared in a number of articles in slightly varying form. Here, we will generally follow the derivation reported in Patterson and Moore (1986) which presents a review and distillation of the previous work.

In its most general form, the power-spectrum model of masking may be written as:

$$P_s = k \int_0^\infty N(f)W(f)\,df, \qquad (1)$$

where P_s is the threshold of the probe tone (in power units), $N(f)$ is the power spectrum level of the noise as a function of frequency f, $W(f)$ is a weighting function representing the squared magnitude of the auditory filter's transfer function, and k is a constant representing the efficiency of the detection process following auditory filtering.

For notched-noise maskers, which can always be thought of as consisting of two bands of noise, $N(f)$ has a constant value of N_0 over certain frequency limits, and is 0 otherwise. Also, as we are assuming that the notch edges are equa-distant from the probe at frequency f_0, we need only concern ourselves with equations of the form:

[1] Young adult normal listeners would typically exhibit p values of about 15 in this condition.

$$P_s = k N_o \int_{f_{lo}}^{f_o - nw/2} W(f)\, df + k N_o \int_{f_o + nw/2}^{f_{hi}} W(f)\, df , \qquad (2)$$

where the first term on the right hand side (RHS) accounts for the lower band of the masking noise, and the second term accounts for the upper band. The extreme frequency limits of the masker are represented by f_{lo} and f_{hi}, while nw represents the total notch width. For the no-notch condition, the upper limit of the first integral and the lower limit of the second are both set to f_o by letting $nw = 0$.

The shapes of the roex family of filters are more conveniently described in terms of a variable known as g, rather than directly in frequency, where:

$$g = |f - f_o| / f_o . \qquad (3)$$

Note that f_o is the frequency of the probe, and hence the centre frequency (cf) of the auditory filter under consideration. Thus g is the distance in frequency from the filter cf, normalized by the cf.

For the first integral on the RHS of (2), $f \le f_o$ whereas for the second integral, $f \ge f_o$. Therefore, we can avoid the use of the absolute value operator by rewriting (2) as:

$$P_s = k N_o \int_{f_{lo}}^{f_o - nw/2} ROEX((f_o - f)/f_o)\, df + k N_o \int_{f_o + nw/2}^{f_{hi}} ROEX((f - f_o)/f_o)\, df , \qquad (4)$$

where ROEX() is one of the roex family of filters substituting for W(f). To convert the variable of integration to g, we note again that for $f > f_o$ as in the second integral of (4):

$$g = (f - f_o)/f_o , \qquad (5)$$

$$f_o = f_o g + f_o , \qquad (6)$$

and taking derivatives with respect to g on both sides of (6) that:

$$df/dg = d(f_o g + f_o)/dg = f_o , \qquad (7)$$

so that:

$$df = f_o\, dg. \qquad (8)$$

Similarly, when $f < f_o$, it is easy to see that:

$$df = - f_o\, dg. \qquad (9)$$

Therefore, (4) becomes:

$$P_s = -k N_o f_o \int_{g_{lo}}^{g_{nw}} ROEX(g)\, dg + k N_o f_o \int_{g_{nw}}^{g_{hi}} ROEX(g)\, dg , \qquad (10)$$

where g_{lo} and g_{hi} are the extreme frequency limits of the masker now expressed in terms of g, and $g_{nw} = nw/2f_o$, the transformed notch width. For frequencies that are less than f_o (first integral on the RHS of equation 10), the transformation to g is monotone decreasing. Therefore, as $f_{lo} < f_o - nw/2$ (in order for there to be a band of noise below the notch at all), so $g_{lo} > g_{nw}$. Since the noise power is positive, so too must the integral be positive, representing the power of the lower noise band passed by the filter. This accounts for the negative sign preceding the first integral. For ease of reading, we can simply reverse the limits of that integral, changing the minus sign to a positive one to obtain:

$$P_s = k N_o f_o \int_{g_{nw}}^{g_{lo}} ROEX(g)\, dg + k N_o f_o \int_{g_{nw}}^{g_{hi}} ROEX(g)\, dg . \qquad (11)$$

This is the most general expression that can be obtained for the roex family of filters under the assumptions of constant noise spectrum level and symmetry of notch and probe. In Patterson and Moore's (1986) derivation, $f_{lo} = 0$ Hz while $f_{hi} = \infty$, leading to $g_{lo} = 1$ and $g_{hi} = \infty$. This is equivalent to assuming that the masker is infinitely wide, extending from 0 Hz upward. Patterson and Moore then go on to argue that, as the filter is symmetric in g, the two integrals in equation (11) are equal, so that equation (11) may be reduced to:

$$P_s = 2 k N_o f_o \int_{g_{nw}}^{\infty} ROEX(g)\, dg \qquad (12) \qquad (12)$$

In fact, it is clear from equation (11) that the two integrals are *not* equal, because $g_{lo} \neq g_{hi}$. The error arises because roex filters are *not* strictly symmetric, but are only symmetric for the frequency range extending from 0 Hz to $2f_o$ Hz. This can be seen most clearly by noting that the upper part of the filter shape ($f > f_o$) extends towards infinity, while the lower part ($f < f_o$) can only extend to 0 Hz and no further, as a filter value for negative frequencies is nonsense in this context. Relatively sharp filters go to zero fast enough in both directions from f_o so that the two integrals in equation (11) are approximately equal, making equation (12) accurate enough for most uses. For shallow filters (as typically occur for hearing-impaired listeners), the RHS of equation (12)

approximates the RIIS of equation (11) rather poorly.

The two integrals in equation (11) can be made equal for appropriate integration limits. As implied above, if $g_{lo} = g_{hi}$, then equations (11) and (12) are equivalent (substituting ghi for the upper limit of the integral in (12)). This condition obtains nearly universally in published empirical studies, and simply indicates that the upper and lower limits of the masker are equa-distant from the probe.

In order to demonstrate more clearly the effects of these various assumptions, we shall now assume use of the simplest member of the roex family, the so-called roex(p) model defined by:

$$\text{roex}(p) = (1+pg)\, e^{-pg} \quad . \tag{13}$$

Calculating the indefinite integral gives:

$$\int (1+pg)\, e^{-pg}\, dg = -p^{-1}(2+pg)\, e^{-pg} \quad . \tag{14}$$

To obtain the predictions of the roex(p) model according to Patterson and Moore, we substitute this expression into (12), to obtain:

$$P_s = -(2k\, N_o\, f_o\, p^{-1})(2+pg)\, e^{-pg} \Big|_{g_{nw}}^{\infty} \quad . \tag{15}$$

Finally, noting that:

$$\lim_{g \to \infty} (2+pg)\, e^{-pg} = 0 \quad , \tag{16}$$

$$P_s = 2k\, N_o\, f_o\, p^{-1}(2+pg_{nw})\, e^{-pg_{nw}} \quad . \tag{17}$$

This is the (incorrect) equation Patterson and Moore give in order to predict the threshold of the probe (in power terms) for a particular notch width. For the case $g_{lo} = g_{hi}$, the use of the appropriate limits of integration makes (12) become:

$$P_s = -(2k\, N_o\, f_o\, p^{-1})(2+pg)\, e^{-pg} \Big|_{g_{nw}}^{g_{hi}} \quad . \tag{18}$$

and:

$$P_s = 2k\, N_o\, f_o\, p^{-1}\left[(2+pg_{nw})\, e^{-pg_{nw}} - (2+pg_{hi})\, e^{-pg_{hi}}\right] . \tag{19}$$

There is thus an extra term in the equation compared to that given by Patterson and Moore. Because this extra term is always positive, the incorrect formula will always predict a threshold greater than the correct one.

The magnitude of the error introduced by the use of equation (17) instead of equation (19) can be examined by looking at the difference (in dB) between the two predictions:

$$\text{Error} = 10\log(P_{s\text{-correct}}) - 10\log(P_{s\text{-wrong}}) \tag{20}$$

$$= 10\log(P_{s\text{-correct}}/P_{s\text{-wrong}}) , \tag{21}$$

where, after cancellation of appropriate terms:

$$P_{s\text{-correct}}/P_{s\text{-wrong}} = \frac{(2+pg_{nw})\, e^{-pg_{nw}} - (2+pg_{hi})\, e^{-pg_{hi}}}{(2+pg_{nw})\, e^{-pg_{nw}}} \tag{22}$$

$$= 1 - \frac{(2+pg_{hi})\, e^{-pg_{nw}}}{(2+pg_{nw})\, e^{-pg_{nw}}} \tag{23}$$

Assuming $g_{hi} = 0.8$ (representative of the values typically used in notched-noise studies), the error as given by equations (20)-(23) was calculated for g_{nw} between 0.0 and 0.4 inclusively. For $p \geq 5$, the maximum error magnitude was always less than 1 dB, decreasing with increasing p. For a fixed p, the magnitude of the error increases with g_{nw}. For example, with $p = 3$, the magnitude of the error at $g_{nw} = 0.0$ was just under 1 dB, while at $g_{nw} = 0.4$, it was 2.3 dB. It thus appears that errors will be small as long as auditory filtering is reasonably narrow band (as indicated by a moderately high p value). As normal listeners display auditory filtering consistent with p of at least 7 (occurring at the lowest frequency tested of 125 Hz - Fidell, Horonjeff, Teffeteller & Green, 1983; Rosen & Stock, 1989), the error is inconsequential for studies of normal hearing, being less than the typical variability in measured thresholds. Note however, that lowering the value of g_{hi} will cause greater errors.

In most applications of the roex(p) model, however, the value of p is estimated from a known set of masked thresholds. Two ways were used to determine the errors in estimating p arising from use of the the incorrect formula.

Firstly, the correct roex(p) formula (equation 19) was used to generate a set of masked thresholds (in dB) for $0 \leq p \leq 20$, $k = .5$ ($10\log k = -3.0$ dB) and g_{nw} equal to 0.0, 0.1, 0.2, 0.3 and 0.4 (a reasonable set to use in notched-noise experiments). Then, for each synthesized data set (from a single value of p), a minimization routine was used to estimate values of p and $10\log k$ (symbolized as p_{est} and $10\log k_{est}$) that best predicted the data using the incorrect formula (equation 17)[2]. For both parameters, the magnitude of the error decreased with increases in p, with $10\log k_{est}$ never being further from $10\log k$ than 0.65 dB. The value of p_{est} could be far from p however, as has already been seen in a previous section. Figure 2 shows for $0 \leq p \leq 15$ the relationship between p and p_{est}. As noted earlier, the error is greatest for low p values. A true p of 1, for

[2]As recommended by Patterson and Moore (1986), all model fits were on the basis of values expressed in dB. Therefore, equations (17) and (19) were converted by taking logarithms (to the base 10) and multiplying by 10. The fitting procedure was implemented using the SAS procedure NLIN which uses a least-squares criterion for minimization. In order to avoid the calculation of partial derivatives, the multivariate secant technique (also known as false position) was used. For further details see the SAS/STAT Guide for Personal Computers 1987.

instance leads to a p_{est} of 3.06, a percentage error of over 200%. As long as $p > 5$, the percentage error is less than 10%, and becomes less than 1% for $p > 9.6$. At p values near 20, the error in p is less than .005%, and on the order of 0.0005 dB for **10logk**. Again, this suggests that the error is relatively inconsequential for normal listeners (where p is almost always greater than 7), but could be serious for hearing-impaired ones.

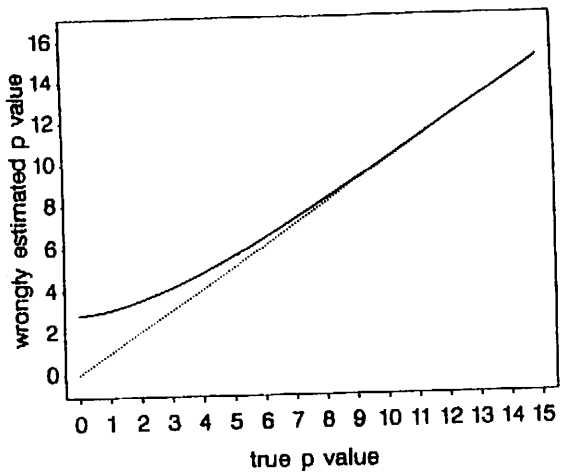

Figure 2. The relationship between the true p used to generate a synthetic data set and the p estimated on the basis of an incorrect formula, both based on a roex(p) filter shape (solid line). The dashed line shows what would be expected if there was no error in estimation.

Secondly, a set of data from normal listeners was analyzed using both the correct and the incorrect formula, in order to estimate the magnitude of errors that might arise in considering genuine data. The analyzed data came from a study of low-frequency selectivity by Rosen and Stock (1989). Five normal listeners were tested at four probe frequencies (125, 250, 500 and 1000 Hz) and four noise levels (40 to 70 dB SPL/Hz in 10 dB steps) at five symmetric notch widths (0.0, 0.1, 0.2, 0.3 and 0.4). The bands of noise were constructed such that g_{hi} was 0.72 for probes at 125 Hz and 0.8 otherwise. Data sets were selected so as to have low p values, with a few high p sets for comparison. As expected, nearly all of the lowest p data sets were for probe frequencies of 125 Hz (the sole exception being at 250 Hz). In agreement with the conclusions derived from synthetic data, errors in p were more serious than errors in **10logk**. Values of **10logk** estimated correctly were, with one exception, within 0.3 dB of the values of **10logk** estimated by use of the incorrect formula. Maximum percentage errors in p were about 5-7%, for data that was fit with correct p values near 7. One set of data, because it indicated a fairly low degree of selectivity, led to much greater errors, with an incorrect p of 3.4, and a correct one of 0.7, a percentage error of 386%. The error in k, too, was greater than for the other data sets but still only about 0.8 dB. Although this data set may not in fact be representative of the selectivity typically displayed by normal listeners, it is a warning that using the wrong integration limits can lead to serious errors, even in normal listeners (at least at low frequencies). For impaired listeners, the errors may be very serious. For normal listeners with probe frequencies at 250 Hz and above, the error in p is

unlikely to reach even a few per cent.

So far, we have only developed formulae for the case $g_{lo} = g_{hi}$. In the most general case, $g_{lo} \neq g_{hi}$, and predictions from the roex(p) model are given by:

$$P_s = kN_0 f_0 p^{-1} [2(2+pg_{nw})\, e^{-pg_{nw}} - (2+pg_{lo})\, e^{-pg_{lo}} - (2+pg_{hi})\, e^{-pg_{hi}}] \qquad (24)$$

Thus, equation (19) can be seen to be a special case of this more general equation.

Other roex filter shapes have been defined. The most complex, the so-called roex(p,w,t) model consists of two rounded exponentials, one of which dominates in the centre of the passband of the filter, and one of which dominates towards the filter tails. Patterson and Moore's (1986) derivation for this model contains the same error as detailed for the roex(p) model, and the correct derivation would follow the same line as given above. As the roex(p,w,t) model has not seen much use (few experiments have the number of data points needed to tightly constrain a model with four free parameters, nor sufficient range to define the filter tails), we omit the correct formula here. However, it is as well to note that since the tails of the filter in this model are typically much shallower than the falloff in the passband, the errors caused by the use of the incorrect formula will be correspondingly greater.

Finally, much use has been made of the so-called roex(p,r) model, given by:

$$\text{roex}(p,r) = (1-r)(1+pg)\, e^{-pg} + r \ . \qquad (25)$$

This is simply the roex(p) shape with an additive constant and scaling factor to keep its value equal to unity at its centre frequency. Errors in using this model are likely to be much less severe than in the other two models because some integration limit less than infinity has to be used in order to keep the integral in equation (12) bounded. Patterson and Moore suggest setting $g_{hi} = 0.8$, a value in keeping with what experimenters have typically used. It seems more sensible simply to use the integration limits appropriate to a particular experiment, rather than setting an arbitrary value. It is easy to imagine situations in which g_{hi} could be greater or smaller than 0.8.[3]

Finally, it is well to note that the error of assuming unbounded filter symmetry in derivations for masking studies has not only been confined to roex filters and notched-noise experiments. Glasberg, Moore and Nimmo-Smith (1984) for example, make the same error in deriving predictions for rippled-noise maskers and Gaussian filters. As that study investigated the selectivity of normal listeners at 1 kHz and low levels, however (where p is typically about 25), it is unlikely that the error in derivations would be important.

[3] In the model fits used here, the experimental procedure was such that g_{lo} and g_{hi} were not only equal, but constant over notch widths. Such a situation arises when a single band-pass noise is modulated by a sinusoid at f_0 in order to create the notched noise (e.g., as used by Weber, 1977; Rosen & Stock, 1989). In other paradigms, two separate low-pass bands of noise are modulated independently to create the notched noise (e.g., Patterson 1976), making g_{lo} and g_{hi} vary with notch width. In that case too, the true limits of the noise should be used as the integration limits, a task which requires little extra programming work.

4. The calculation of a bandwidth parameter

In reporting results from roex modelling of masking experiments, it is relatively rare to see p values alone directly quoted. Instead, a more generally understood measure of bandwidth is typically given, the *equivalent rectangular bandwidth* (BW_{ER} - also known as equivalent noise bandwidth). The BW_{ER} of any filter is defined as the bandwidth of an ideal bandpass filter (i.e., with cutoffs of infinite slope) which would pass the same power of white noise as the original filter, assuming the efficiency of transmission for the ideal rectangular filter to be equal to the maximal value of the filter under consideration. Because the three roex filters have unity gain at their peak:

$$BW_{ER} = \int_0^\infty ROEX(|f_o - f|/f_o)\, df$$

$$= \int_0^{f_o} ROEX((f_o - f)/f_o)\, df + \int_{f_o}^\infty ROEX((f - f_o)/f_o)\, df$$

$$= f_o \int_0^1 ROEX(g)\, dg + f_o \int_0^\infty ROEX(g)\, dg \quad . \qquad (26)$$

(following roughly the same steps as in the previous section, with $f_{lo} = 0$, $f_{hi} = \infty$ and nw=0). For the roex(p) filter, this becomes:

$$BW_{ER} = \frac{4f_o}{p} - \frac{f_o(2+p)\,e^{-p}}{p} \qquad (27)$$

Patterson and Moore (1986) give the BW_{ER} of this filter as $4f_o/p$. This discrepancy arises from the same error discussed in the previous section - that of assuming the roex filter is symmetric for unbounded g. The BW_{ER} that Patterson and Moore report results from substituting $g_{nw} = 0$ into equation (12), and then taking the integral of the roex(p) filter. Again the error is inconsequential for normal selectivity as can be seen by calculating the percentage error between these two values:

$$\%ERROR = \frac{(BW_{ER:CORRECT} - BW_{ER:WRONG})}{BW_{ER:CORRECT}} \times 100 \ ,$$

$$= \frac{-(2+p)\,e^{-p}}{4 - (2+p)\,e^{-p}} \times 100 \qquad (28) \qquad (28)$$

Figure 3 shows the percentage error as a function of p. For $p \geq 6$, the error is less than 1%, but increases sharply with decreasing p, with a 20% error for p values just below 2. Again, these errors can be serious for degraded auditory filtering.

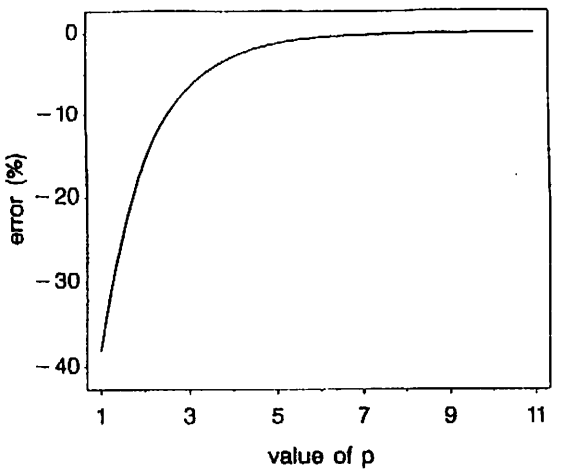

Figure 3. The percentage error in equivalent rectangular bandwidth for a roex(p) filter calculated by a correct and an incorrect formula.

Perhaps more disturbing, equivalent rectangular bandwidths are quoted for roex filters which do not have a finite BW_{ER} - to wit, roex(p,r) filters. As noted in the previous section, roex(p,r) filters are the sum of a roex(p) filter plus a constant. Therefore, their BW_{ER}s are infinite. Yet Moore and Glasberg (1983), in summarizing the outcomes of 6 studies using roex(p,r) filters, give all their results in terms of equivalent rectangular bandwidth. In fact, the BW_{ER} used in that paper is simply $4f_o/p$, an approximation to BW_{ER} for roex(p) filters.

At least four alternatives are available for reporting a measure of bandwidth. Firstly, one can continue to quote BW_{ER}'s, but in reference to roex(p,r) filters to use some sort of circumlocution such as "the equivalent rectangular bandwidth of a roex(p) filter with p derived from a roex(p,r) filter". This is perhaps the least desirable alternative. Secondly, one can take the route suggested by Patterson and Moore for the derivation of thresholds from the roex(p,r) model by setting a finite upper integration limit in equation (26). A value of $g_{hi} = g_{lo} = 1$, might be preferable to $g_{hi} = g_{lo} = 0.8$, in that it represents the maximum extent to which the roex family of filters is symmetric ($g_{lo} = 1$ corresponding to a lower integration limit of 0 Hz). It can then be shown that this modified equivalent rectangular bandwidth is given by:

$$BW_{ER:MOD} = 2f_o\,[p^{-1}\,(1-r)(2-(2+p)\,e^{-p}) - r] \qquad (29) \qquad (29)$$

The advantage of this formula over simply using the BW_{ER} from a roex(p) filter is that it depends on r as well as p, as clearly any approximation to an equivalent rectangular bandwidth for a roex(p,r) filter must.

Thirdly, one can report p values directly, although the meaning of such numbers is obscure to the uninitiated. Finally, one can calculate 3-dB bandwidths (as has been done, for example, by Glasberg, Moore & Nimmo-Smith, 1984). This has the advantage of characterizing the properties of the auditory filter in its main passband, something which a true equivalent rectangular bandwidth does not. For the roex(p) filter, the 3-dB bandwidth (BW_{3dB}) may be derived as follows. Because a 3-dB change is equivalent to a reduction in intensity of a factor of 2, we first solve for g_{3dB} in:

$$(1+pg_{3dB})\,e^{-pg_{3dB}} = 0.5 \ , \qquad (30)$$

where g_{3dB} represents the normalized upper frequency limit of the 3-dB passband of the filter.

As of g_{3dB} and p always enter equation (30) as a product, and never separately, this is equivalent to solving the following equation for x:

$$(1+x)\,e^{-x} - 0.5 = 0 \ . \qquad (31)$$

As such an equation cannot be solved in closed form, we use instead a numerical root-finding algorithm to find that $x \approx 1.6783$. Clearly then:

$$pg_{3dB} \approx 1.6783 \ , \qquad (32)$$

or:

$$g_{3dB} \approx \frac{1.6783}{p} \ . \qquad (33)$$

The value of g_{3dB} is converted back into frequency using equation (6). This f_{3dB} will then represent the frequency value at which the filter is 3 dB down from its peak on the upper frequency side. In order to calculate 3-dB bandwidth, f_o must be subtracted from it, and this then needs to be multiplied by 2:

$$BW_{3dB} = 2f_o\,g_{3dB} \qquad (34) \qquad (34)$$

Finally, substituting (33) into (34) gives:

$$BW_{3dB} \approx \frac{3.357\,f_o}{p} \ . \qquad (35)$$

Recall however, that $4f_o/p$ is an approximation to the equivalent rectangular bandwidth for a roex(p) filter. As both $BW_{ER:WRONG}$ and BW_{3dB} are proportional to $1/p$, they are clearly proportional to one another, with $BW_{ER:WRONG}$ being about 20% larger than BW_{3dB}. Figure 4 shows how the three measures of bandwidth vary as a function of p for a roex(p) filter.

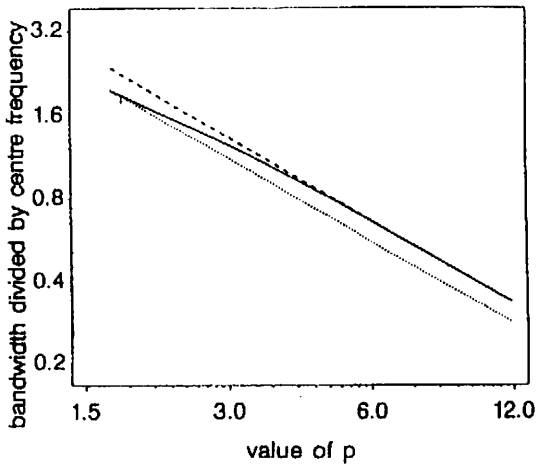

Figure 4. Three different measures of relative bandwidth for the roex(p) model, each as a function of p. The upper (dashed) line gives the $BW_{ER:WRONG}$, while the lower (dotted) line gives 3-dB bandwidth. The middle (solid line) gives $BW_{ER:CORRECT}$.

Using 3-dB bandwidths has two slight drawbacks. Firstly, as argued in the previous section, g should not be allowed to become greater than 1. Thus, for a roex(p) filter, this limits the calculation of a 3-dB bandwidth to filters with $p \geq 1.678$. Secondly, numerical root-finding algorithms are necessary to calculate 3-dB bandwidths, albeit very simple ones. The advantages, however, are many. The concept is the most commonly-used one for characterizing filters, and is applicable to all members of the roex family (in fact, all filters that are not too wide). For the roex(p,r) filter, r will have little effect on the 3-dB bandwidth as long as it is small enough. It will therefore reflect selectivity primarily in the central pass band. For these reasons, it is recommended that experimenters report both the estimated parameters and 3-dB bandwidth, discontinuing use of BW_{ER} as it has most frequently been used for filter shapes to which it is inapplicable.

5. Summary

It has been shown that the formulae given by Patterson and Moore (1986) for fitting detection thresholds to roex filter shapes, and for calculating equivalent rectangular bandwidths are in error. The effect of these errors is small for the vast majority of studies on normal hearing listeners but may be serious for studies with hearing-impaired listeners, or moderately serious for normal listeners in the low-frequency range. Errors in estimation can be completely avoided by the use of appropriate limits of integration in derivations, limits which represent the frequency extent of the notched noise. The resulting formulae are only marginally more complex than the simplified approximate ones, and represent a trivial increase in programming and computing time. Limitations in the use of equivalent rectangular bandwidth have been pointed out, and the use of a 3-dB bandwidth suggested instead.

6. Acknowledgements

I am grateful to Brian Glasberg and Brian Moore for allowing me access to their original computer programs for fitting roex filters to detection thresholds. David Smith made useful comments on the manuscript and also exercised his typesetting skills upon it. This work was supported by the Medical Research Council of Great Britain.

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Note: These corrections have already been inserted into the text in the appropriate places.

ERRATA FOR:

DERIVING AUDITORY FILTER CHARACTERISTICS FROM NOTCHED-NOISE MASKING DATA: MODIFIED DERIVATIONS

Stuart ROSEN

which appeared in Speech, Hearing and Language: Work in Progress (1989) Volume 3, pages 189-204.

A number of equations in the above-named paper in the last progress report were unfortunately found to be in error. The correct equations follow.

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pg. 195:

$$P_s = 2\,k\,N_o\,f_o \int_{g_{nw}}^{\infty} ROEX(g)\,dg \qquad (12)$$

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pg. 199:

$$P_s = kN_o f_o p^{-1}\left[2(2+pg_{nw})\,e^{-pg_{nw}} - (2+pg_{lo})\,e^{-pg_{lo}} - (2+pg_{hi})\,e^{-pg_{hi}}\right] \qquad (24)$$

---

pg. 200:

$$= \frac{-(2+p)\,e^{-p}}{4-(2+p)\,e^{-p}} \times 100 \qquad (28)$$

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pg. 201:

$$BW_{ER:MOD} = 2f_o\left[p^{-1}\,(1-r)(2-(2+p)\,e^{-p}) - r\right] \qquad (29)$$

---

pg.202

$$BW_{3dB} = 2f_o\,g_{3dB} \qquad (34)$$

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